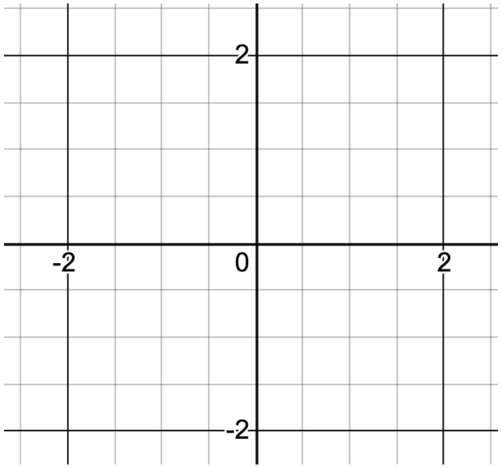
Chapter 13: Vectors Functions

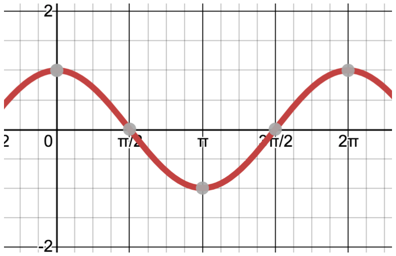
13.1: Vector Functions and Space Curves

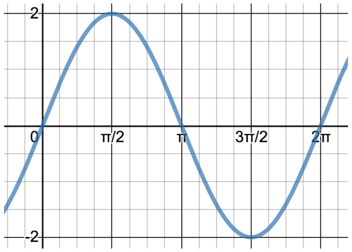
Parametric Equations in R^2



Graph: $\begin{cases} x = 2\sin t \\ y = \cos t \end{cases}$ $\Rightarrow$ Eliminate the parameter

Alternatively, can use info from x(t) and y(t) graphs.





Written in vector form: $\vec{r}(t) = \langle 2\sin t, \cos t \rangle$. We define this as a ___________________________ and think of the graph as being traced out by ____________________________

Parametric Equations in R^3

Here we consider parametric equations and vector functions in R^3. A curve in R^3, "space curve" can be expressed parametrically

$$\begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}$$ or as a vector valued function of the form $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

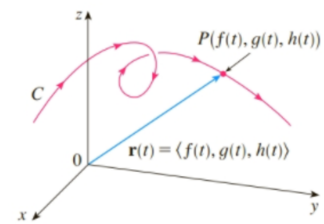
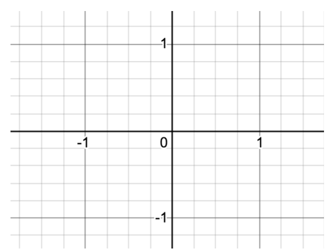


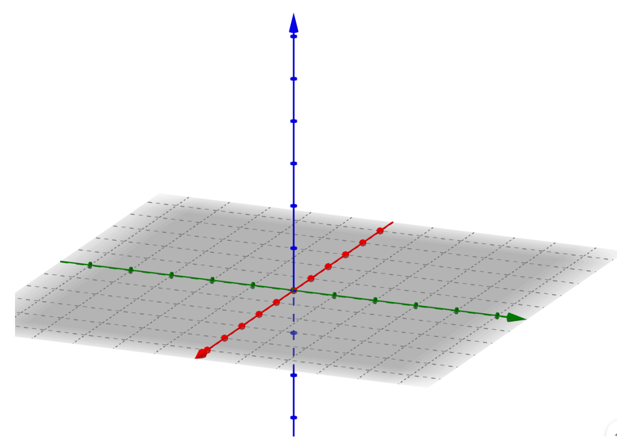
FIGURE 1
C is traced out by the tip of a moving position vector **r**(*t*).

See animation of 5C page https://www.geogebra.org/m/RtISr7GW#material/Tsbi3UY9

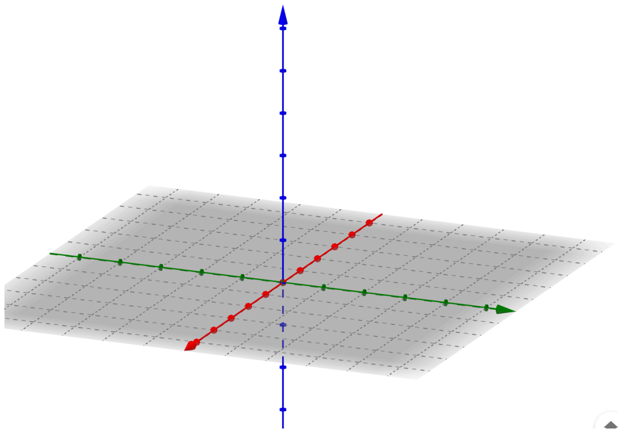
Sketching Space Curves

Example: $\vec{r}(t) = \langle \cos(t), \sin(t),\ t \rangle$

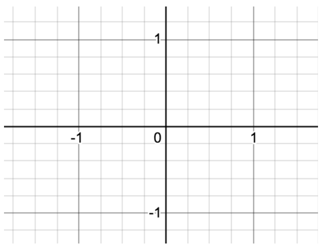


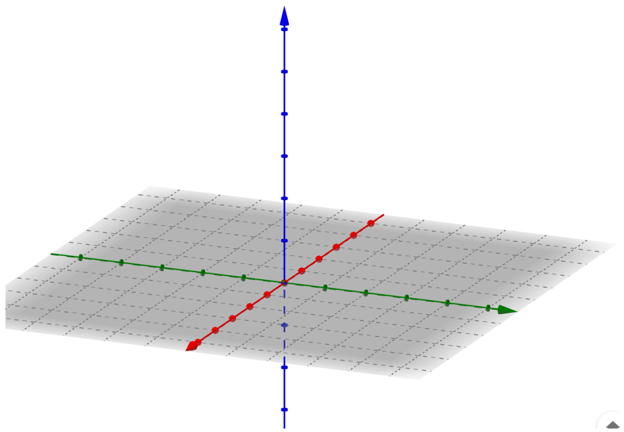


Example: $\vec{r}(t) = \langle \cos(t), \sin(t),\ e^{t} \rangle$



Example: $\vec{r}(t) = \langle \sin(t), \cos(t),\ t \rangle$



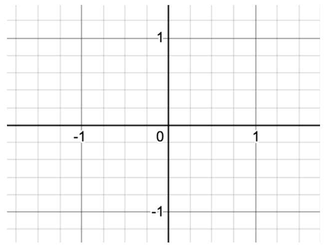


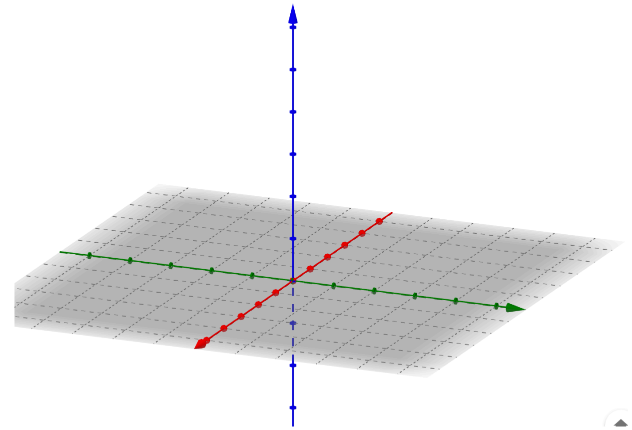
When graphing space curves, you will be required to show

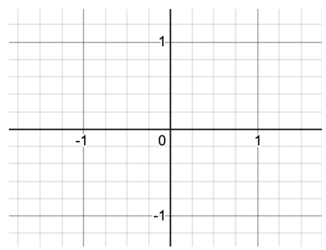
____ and to clearly show

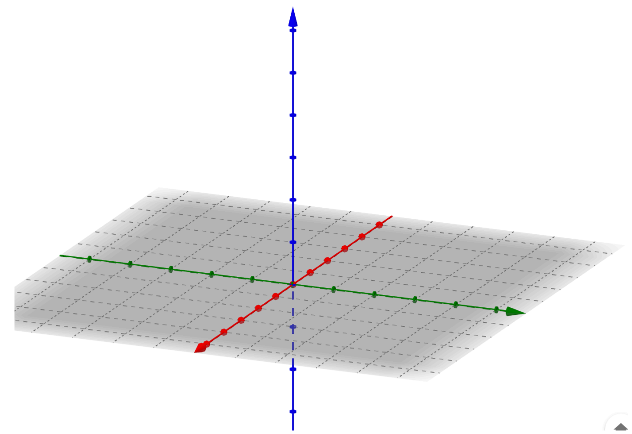
____ and ____________________________

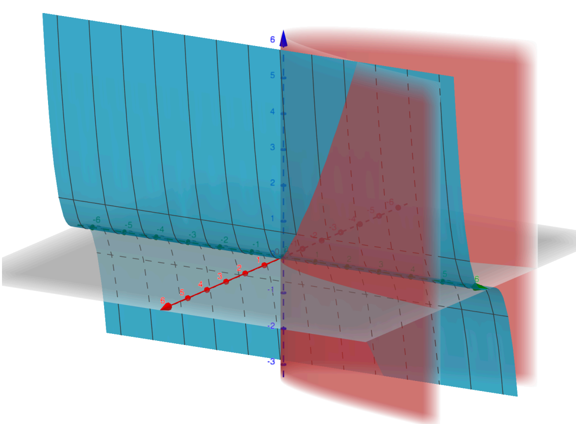
Example: $\vec{r}(t) = \left\langle t,\ t^2,\ t^3 \right\rangle$



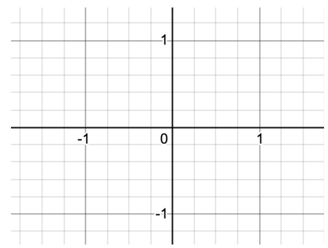


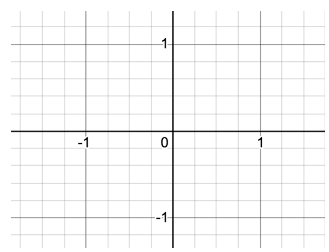


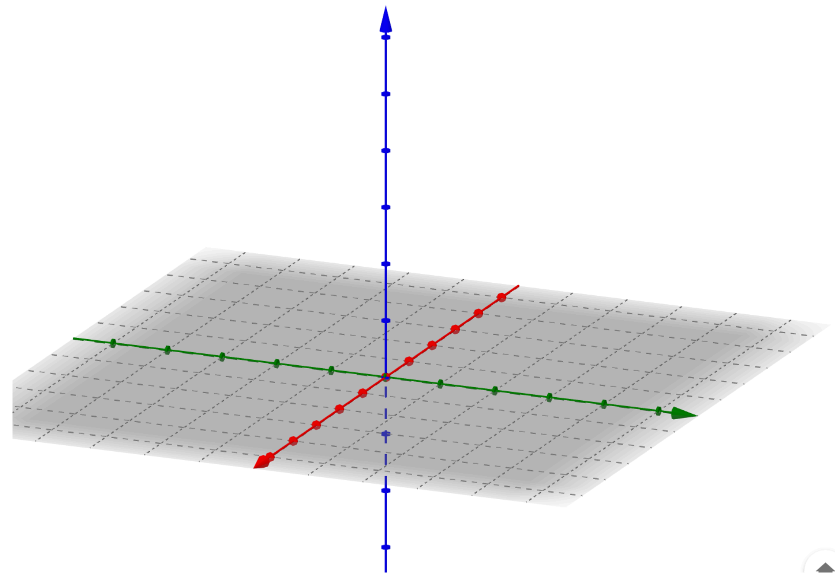




Example: $\vec{r}(t) = \left\langle \cos t, \cos t, \sqrt{2}\sin t \right\rangle$







Parameterizing a surface

Find the vector valued function to represent the curve of intersection of $x^2+y^2=4$ and $z=xy$

Further study of Vector Valued Functions

Domain:

Limit:

Continuity:

13.2: Derivatives and Integrals of Vector Valued Functions

Derivatives

Recall: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

Similarly we will define $\vec{r}\,'(t) = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0}$ ____________________

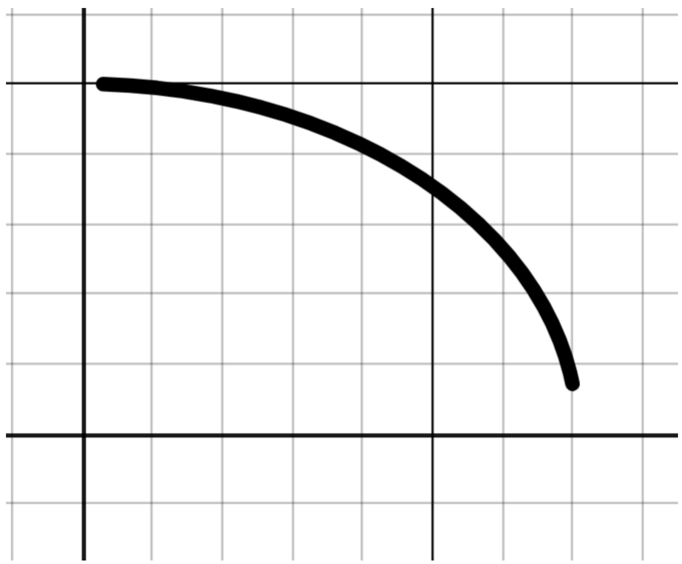
How do we compute this and what does it give us geometrically ?

$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ then $\vec{r}\,'(t) = \lim_{\Delta t \to 0} \frac{\langle f(t+\Delta t), g(t+\Delta t), h(t+\Delta t) \rangle - \langle f(t), g(t), h(t) \rangle}{\Delta t} =$

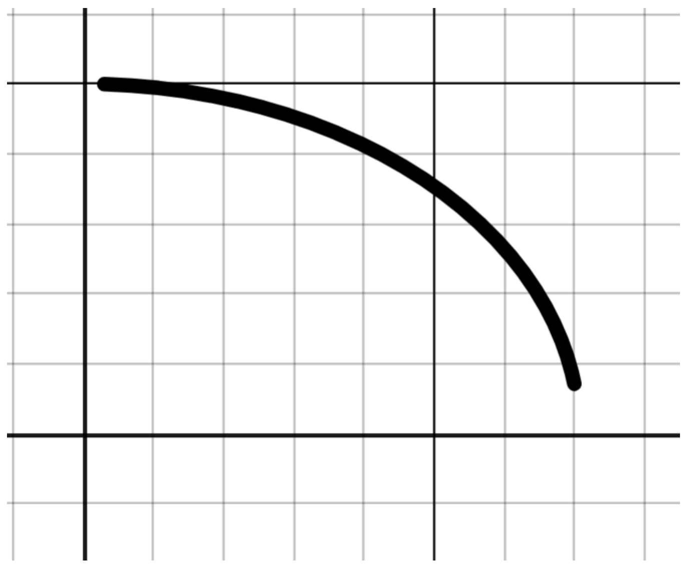
Example: If $\vec{r}(t) = \langle \cos t, \ln t, t^3 \rangle$, find $\vec{r}\,'(t)$

Geometric Meaning of $\vec{r}\,'(t)$

$\Delta t > 0$



$\Delta t < 0$



So $\vec{r}\,'(t)$ is ______________________________ to the curve C at the tip of $\vec{r}(t)$ and in the direction of ______________________

Tangent vector fails to exist at t_0 if (1) $\vec{r}\,'(t_0)$ fails to exist or (2) $\vec{r}\,'(t_0) = \vec{0}$. C is called _______________________ if $\vec{r}\,'(t)$ is continuous and if $\vec{r}\,'(t\ \ ) \neq \vec{0}$. We are often asked to find the unit tangent vector, $\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$

Example: Find equations of the line tangent to $\vec{r}(t) = \langle \cos(t), \sin(t),\ t \rangle$ at t=π/6.

Integrals

We define the definite integral of a vector valued function as

$$\int_a^b \mathbf{r}(t)\, dt = \lim_{n \to \infty} \sum_{i=1}^{n} \mathbf{r}(t_i^*)\, \Delta t$$

Which leads to computation

$$\int_a^b \mathbf{r}(t)\, dt = \left(\int_a^b f(t)\, dt\right)\mathbf{i} + \left(\int_a^b g(t)\, dt\right)\mathbf{j} + \left(\int_a^b h(t)\, dt\right)\mathbf{k}$$

Example-Indefinite Integral : Compute $\int \vec{r}(t)dt$, for $\vec{r}(t) = \langle \cos(t), \sin(t),\ t \rangle$

13.4 Application- Projectile Motion

EXAMPLE 6 A projectile is fired with muzzle speed 150 m/s and angle of elevation 45° from a position 10 m above ground level. Where does the projectile hit the ground, and with what speed?